B. Sc (Hons) MATHS

2009- 2010

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FACULTY OF MATHEMATICAL SCIENCES
UNIVERSITY OF DELHI
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No. MF/2008/704
Dated : 12.12.2008

Prof. (Ms.) Manju Lata Agarwal
Dean

Recd 22/12/08

29. The Principal
Jesus & Mary College
Chanakyapuri
New Delhi- 1100 21.

Subject: **Revised syllabus of B.Sc.(Hons.) Mathematics for implementation w.e.f. the Academic Session 2009-2010.**

Dear Sir/Madam,

I am enclosing herewith a copy of the revised syllabus of B.Sc (Hons.) Mathematics duly approved vide Academic Council Resolution No. 15 dated 12th July 2008 and Executive Council Resolution No. 56(10) dated 29th July 2008 for implementation from the Academic Session 2009-2010.

It is requested that the same may kindly be circulated to all the teachers of Department of Mathematics in your college.

Thanking you,

Yours faithfully,

M L Agarwal
12.12.08
DEAN

Part 'A'
A.C. 12.07.2008
Item No. 2.1.7
Annexure No.XXIV

Revised Syllabus for B.Sc (Hons) Mathematics

There will be a restructured programme in BA (Hons.)/ B.Sc (Hons.) Mathematics. The new restructured programme will be known as B.Sc (Hons.) Mathematics. The admission criterion for the new restructured programme will remain the same as for the previous BA (Hons.)/ B.Sc (Hons.) Mathematics.

The restructured B.Sc (Hons.) Mathematics will have 12 papers of mathematics and five papers from disciplines other than Mathematics. Each paper of mathematics will be of 100 marks and have an examination of 3 hours.

Structure of the course

(a) B.Sc (Hons) Mathematics I year

Paper I: Calculus
Paper II: Analysis I
Paper III: Algebra I

Along with the above mentioned papers, a student will have to opt for three courses from disciplines other than Mathematics. A student will have to choose one course each from Credit Course I, Credit Course II and Qualifying Course. The marks of Credit Course I and Credit Course II shall count in the final result of the student.

Credit Course I

- (i) Ethics in Public Domain
- (ii) Environmental Issues in India
- (iii) Reading Gandhi
- (iv) The Individual and Society
- (v) Hindi Language, Literature and Culture
- (vi) Gender and Society
- (vii) Financial Management
- (viii) Chemistry
- (ix) Physics-I

Note: (a) Courses (i)-(vi) are the interdisciplinary courses of the BA (Hons) Programme.
(b) Course (vii) is the elective course EL 210 (vi) of B.Sc Programme.
(c) Course (viii) is the Paper V being taught in First year Physics (Hons).
(d) Course (ix) is the Paper V being taught in First year Chemistry (Hons).

Credit Course II

- (i) English
- (ii) Hindi
- (iii) Sanskrit

(iv) Chemistry
(v) Physics-I

Note: (a) Courses (i)-(iii) are the language credit courses of the BA (Hons) Programme.
(b) Course (iv) is the Paper V being taught in First year Physics (Hons)
(c) Course (v) is the Paper V being taught in First year Chemistry (Hons)

Qualifying Course

(i) English (Higher)
(ii) English (Lower)
(iii) Hindi (Higher)
(iv) Hindi (Lower)
(v) Sanskrit
(vi) Chemistry (Lab)
(vii) Physics (Lab)

Note: (a) Courses (i)-(v) are the language qualifying courses of the BA (Hons) Programme.
(b) Course (vi) is the Paper VIII being taught in First year Physics (Hons). For students of Maths (Hons), only six out of the twelve experiments will have to be done. These experiments may be selected at the college level.
(c) Course (vii) is the Lab II being conducted in First year Chemistry (Hons)

MARKS:

* Each of the Credit I, Credit II and Qualifying courses are of 50 marks: annual examination 38 marks, internal assessment 12 marks.
* The pass mark for the credit courses is 40 percent.
* The pass mark for the qualifying courses is 36 percent. A student has to pass in the qualifying course to be eligible for an Honors degree. However, the marks in this course will not be counted in the final division awarded.
* Internal assessments will be held for the credit courses but not for the qualifying course.

NUMBER OF LECTURES:

* Two hours per week or two classes for Credit Course I (i)-(vii), Credit Course II (i)-(iii) and Qualifying Course (i)-(v). For each of the credit courses, one tutorial will be held fortnightly for students.
* Three hours per week or three classes for Credit Course I (viii)-(ix), Credit Course II (iv)-(v) and Qualifying Course (vi)-(vii).

RULES:

* Every student must opt for at least one language. It can either be a credit course or a qualifying course. If they are opting for a language in both the credit as well as the qualifying course then these cannot be the same languages.

* A student offering Chemistry/ Physics-I as Credit Course I will not be allowed to offer the same as Credit Course II.
* A student will be allowed to take Chemistry (Lab)/ Physics (Lab) as a qualifying course if they have opted for Chemistry/Physics-I respectively as a credit course.

(b) B.Sc (Hons) II year

Paper IV: Differential Equations and Mathematical Modeling I
Paper V: Analysis II
Paper VI: C++ Programming and Numerical Methods
Paper VII: Algebra II

Along with the above mentioned mathematics paper a student will have to opt for two courses from Credit Course III. These courses are from disciplines other than mathematics. In those subjects where more than one course is offered, the student shall opt for one of the course. The marks of Credit Course III shall count in the final result of the student.

<u>Credit Course III</u>

(i) Psychology for Living
(ii) Hindi Literature
(iii) Modern Indian Literature. Poems and Short Stories: Novel or Play
OR
Cultural Diversity. Linguistic Plurality and Literary Traditions in India.
(iv) Formal Logic/ Symbolic Logic
OR
Readings in Western Philosophy
OR
Theory of Consciousness
(v) Citizenship in Globalizing World
(vi) Culture in India: a Historical Perspective
OR
Delhi: Ancient, Medieval and Modern
OR
Religion and Religiosity in India
OR
Inequality or Difference in India
(vii) Sociology of Contemporary India
(viii) Principles of Geography
OR
Geography of India
(ix) Principles of Economics
(x) Financial Accounting
(xi) Green Chemistry
(xii) Biotechnology
(xiii) Physics- II
(xiv) Biophysics

Note: (a) Courses (i)-(ix) are the discipline centred courses of the BA (Hons) Programme.

(b) Course (x-xii) are the elective courses EL 210 (v), EL 310 (i) and EL 3 (iii) of B.Sc Programme.

(c) Course (xiii) is the Paper XI being taught in Second year Chemistry (H

(d) Course (xiv) is the Paper XXII (Option 2) being taught in Third year Physics (Hons).

MARKS:

* Each course carries 50 marks: annual examination 38 marks, internal assessme 12 marks.
* The pass mark is 40 percent.

NUMBER OF LECTURES:

* Two hours per week or two classes for courses (i-xii). For each of these courses one tutorial will be held fortnightly for students.
* Three hours per week for courses (xiii-xiv).

(c) B.Sc (Hons) III year

Paper VIII: Differential Equations and Mathematical Modeling II
Paper IX: Probability and Statistics
Paper X: Algebra III
Paper XI: Analysis III
Paper XII: Optional (one of the following)
(i) Applications of Algebra
(ii) Discrete Mathematics
(iii) Mathematical Finance
(iv) Number Theory and Cryptography
(v) Optimization

Note: Each college will offer at least two optional courses

Paper I: Calculus

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures. 1 Tutorial (per week per student)

Review of inverse functions, hyperbolic functions, higher order derivatives, Leibniz rule and its applications, concavity and inflection points, asymptotes, l'Hopital's rule, optimization procedure, Fermat's principle of optics and Snell's law, applications in business, economics and life sciences.

References:

[1]: Chapter 1 (Section 1.3), Chapter 4 (Sections 4.3-4.7 (from page 124)).
[2]: Chapter 3 (page 197), Chapter 7 (Section 7.8 (pages 509-512)).
[3]: Chapter 2 (Exercise 26, page 205).

Review of coordinates and vectors in R^3, graphs of spheres and cylinders in R^3, triple product, parametric equations, parameterizing a curve, lines in R^3, forms of equations of a plane in R^3, introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions, tangent and normal components of acceleration, modeling ballistics and planetary motion, Kepler's second law.

References:

[1]: Chapter 9 (Sections 9.1, 9.3 (up to page 468), 9.4-9.5), Chapter 10.

Review of definition of conic sections, standard forms of parabolas, ellipses and hyperbolas, reflection properties of conic sections, rotation of axes and second degree equations, classification into conics using the discriminant, polar equations of conics, sketching conics in polar coordinates, traces of surfaces, quadric surfaces, techniques of graphing quadric surfaces, translation of quadric surfaces, reflections of surfaces in 3-spaces, technique for identifying quadric surfaces.

References:

[2]: Chapter 11 (Sections 11.4-11.6), Chapter 12 (Section 12.7).

Functions of several variables, level curves and surfaces, graphs of functions of two variables, limits and continuity of functions of two and three real variables, partial differentiation (two variables), partial derivative as a slope, partial derivative as a rate, higher order partial derivatives (notion only), equality of mixed partials, tangent planes, approximations and differentiability, sufficient condition for differentiability (statement only), chain rule for one and two independent parameters, illustration of chain rule for a function of three variables with three independent parameters, directional derivatives and the gradient, extrema of functions of two variables, least squares approximation of data, method of Lagrange multipliers, constrained optimization problems, Lagrange multipliers with two parameters.

References:

[1]: Chapter 11.

Singular points, curvature, cylindrical and spherical coordinates, equations of surfaces in cylindrical and spherical coordinates, curve tracing in Cartesian coordinates, curve tracing in polar coordinates.

References:

[1]: Chapter 4 (Section 4.4 (page 145)).
[2]: Chapter 4 (Section 4.3), Chapter 11 (Section 11.1), Chapter 12 (Section 12.8), Chapter 13 (Section 13.5).

Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin^n x\, dx$, $\int \cos^n x\, dx$, $\int \tan^n x\, dx$, $\int \sec^n x\, dx$, $\int (\log x)^n\, dx$, $\int \sin^n x \cos^m x\, dx$, volumes by slicing, disks and washers methods, volumes by cylindrical shells, arc length, arc length of parametric curves, finding arc length by numerical methods, surface of revolution, surface area, work, modeling fluid pressure and force, modeling the centroid of a plane region, volume theorem of Pappus.

References:

[2]: Chapter 8 (Sections 8.2-8.3 (pages 532-538)).
[3]: Chapter 5 (Sections 5.2-5.6, 5.8-5.10).

Use of computer aided software for example, Matlab/ Mathematica/ Maple/ MuPad/ wxMaxima in identifying the singular points, points of inflection and tracing of curves.

References:

1. M. J. Strauss, G. L. Bradley and K. J. Smith, *Calculus* (3rd Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.

2. H. Anton, I. Bivens and S. Davis, *Calculus* (7th Edition), John Wiley and Sons (Asia) Pte. Ltd., Singapore, 2002.

3. G. B. Thomas, Jr. and R. L. Finney, *Calculus: Calculus and Analytic Geometry* (9th Edition), Pearson Education, India, 2005.

Paper II: Analysis I

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures. 1 Tutorial (per week per student)

The algebraic and order properties of R, suprema and infima, the completeness property of R, the Archimedean property, density of rational numbers in R. characterization of intervals, neighborhoods, open sets, closed sets, limit points of a set, isolated points, closure, complements, nested intervals, Cantor intersection theorem for nested intervals, uncountability of R.

References:

[1]: Chapter 2 (Sections 2.1-2.4, 2.5 (up to 2.5.4)), Chapter 11 (Section 11.1 (up to 11.1.6 and 11.1.8)).

Sequences, limit of a sequence, convergent sequences, limit theorems, monotone sequences, monotone convergence theorem, subsequences, convergence and divergence criteria, existence of monotonic subsequences, Bolzano-Weierstrass theorem for sequences and sets, definition of Cauchy sequence, Cauchy's convergence criterion, limit superior and limit inferior of a sequence.

References:

[1]: Chapter 3 (Sections 3.1-3.6).
[3]: Chapter 2 (Sections 10.6-10.7).

Definition of infinite series, sequence of partial sums, convergence of infinite series, Cauchy criterion, absolute and conditional convergence, convergence via boundedness of sequence of partial sums, tests of convergence: comparison test, limit comparison test, ratio test, Cauchy's nth root test (proof based on limit superior), integral test (without proof), alternating series, Leibniz test.

References:

[2]: Chapter 9 (Sections 9.1, 9.18).
[3]: Chapter 2 (Sections 14.9-14.10, 15).

Limits of functions, sequential criterion for limits, divergence criteria, review of limit theorems and one-sided limits, continuous functions, sequential criterion for continuity, discontinuity criterion, Dirichlet's nowhere continuous function (illustrations), combinations of continuous functions and compositions of continuous functions, continuous functions on intervals, boundedness theorem, the maximum-minimum theorem, location of roots theorem, Bolzano's intermediate value theorem, intermediate value property, preservation of interval property.

References:

[1]: Chapter 4 (Sections 4.1-4.3), Chapter 5 (Sections 5.1-5.3).

Uniform continuity, uniform continuity theorem, differentiation, derivative, combinations of differentiable functions, Caratheodory theorem, chain rule, derivative of inverse functions, interior extremum theorem, intermediate value property for derivatives (Darboux's theorem), review of Rolle's theorem, mean value theorem, Cauchy's mean value theorem.

References:

[1]: Chapter 5 (Section 5.4 up to 5.4.3), Chapter 6 (Sections 6.1-6.2, 6.3.2).

Taylor's theorem with Lagrange and Cauchy form of remainders, binomial series theorem, Taylor series, Maclaurin series, expansions of exponential, logarithmic and trigonometric functions, convex functions, applications of mean value theorems and Taylor's theorem to monotone functions.

References:

[1]: Chapter 6 (Sections 6.4 (up to 6.4.6)), Chapter 9 (Section 9.4 (page 271)).
[3]: Chapter 5 (Sections 31.6-31.8).

Use of computer aided software for example. Matlab/ Mathematical/ Maple/ MuPad/ Maxima for Taylor and Maclaurin series of sin x, cos x, log (1+x), e^x, $(1+x)^n$, maxima and minima, inverse of graphs.

References:

1. R. G. Bartle and D. R. Sherbert. *Introduction to Real Analysis* (3rd Edition), John Wiley and Sons (Asia) Pte. Ltd., Singapore, 2002.

2. Sudhir R. Ghorpade and Balmohan V. Limaye, *A Course in Calculus and Real Analysis*. Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2006.

3. K. A. Ross, *Elementary analysis: the theory of calculus*, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.

Paper III: Algebra I

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Directed complex numbers, geometrical representation of addition and subtraction of directed complex numbers, De Moivre's theorem for rational indices and its applications, complex roots of unity, the Argand diagram, circles and straight lines, statement of the fundamental theorem of algebra and its consequences, results about occurrence of repeated roots, irrational roots and complex roots, Descarte's rule of signs, relation between roots and coefficients for any polynomial equation.

References: [4].

Sets, binary relations, equivalence relations, congruence relation between integers, finite product of sets, functions, composition of functions, bijective functions, invertible functions, introduction of finite and infinite sets through correspondence, binary operations, principle of mathematical induction, well-ordering property of positive integers, division algorithm, statement of fundamental theorem of arithmetic.

References:

[1]: Chapter 0.
[2]: Chapter 2 (Sections 2.1-2.4), Chapter 3, Chapter 4 (Section 4.4 up to Definition 4.4.6).

Symmetry of a square, dihedral groups, definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), elementary properties of groups, subgroups and examples of subgroups, centralizer, normalizer, center of a group, cyclic groups, generators of cyclic groups, classification of subgroups of cyclic groups.

References:

[1]: Chapters 1, Chapter 2, Chapter 3 (including Exercise 20 on page 66 and Exercise 2 on page 86), Chapter 4, Chapter 5 (till the end of Example 3).

Definition and examples of rings, properties of rings, subrings, integral domains, definition and examples of fields.

References:

[1]: Chapter 12, Chapter 13 (till the end of Example 10).

Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation Ax = b, solution sets of linear systems, applications of linear systems, linear independence, introduction to linear transformations, the matrix of a linear transformation, inverse of a matrix, characterizations of invertible matrices, partitioned matrices, subspaces of R^n, bases and dimension of subspaces of R^n.

References:

[2]: Chapter 1 (Sections 1.1-1.9) and Chapter 2 (Sections 2.1-2.4, 2.8-2.9).

Vector Spaces over R or C, the vector space C^n over C, subspaces, null space, column space, linear transformations, linearly independent sets and basis, coordinate systems, dimension, rank, change of basis, application to Markov chains, eigenvalues and eigenvectors, characteristic equation.

References:

[2]: Chapter 4 (Sections 4.1-4.7, 4.9), Chapter 5 (Sections 5.1-5.2).

Using computer aided software for example, Matlab/ Mathematica/ Maple/ MuPad/ Maxima for operations of complex numbers, plotting of complex numbers, matrices, operations of matrices, determinant, rank, eigenvalue, eigenvector, inverse of a matrix, solution of system of equations.

References:

1. Joseph A. Gallian, *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi, 1999.

2. Edgar G. Goodaire and Michael M. Parmenter, *Discrete Mathematics with G[illegible] Theory* (2nd Edition), Pearson Education (Singapore) Pte. Ltd., Indian Re[illegible] 2003.

3. David C. Lay, *Linear Algebra and its Applications* (3rd Edition), P[illegible] Education Asia, Indian Reprint, 2007.

4. *Complex numbers and theory of equations*, lecture notes published by the Ins[illegible] of Life Long Learning, University of Delhi, Delhi, 2008.

Paper IV: Differential Equations and Mathematical Modeling I

Total marks: 100
Theory: 70
Internal assessment: 30 (15 for theory and 15 for practicals)
4 Lectures, 1 Practical, 1 Tutorial (per week per student)

Differential equations and mathematical models, order and degree of a differential equation, exact differential equations and integrating factors of first order differential equations, reducible second order differential equations, general solution of homogeneous equation of second order, principle of superposition for a homogeneous equation, wronskian, its properties and applications, application of first order differential equations to acceleration-velocity model, growth and decay model, applications of second order differential equations to mechanical vibrations.

References:

[2]: Chapter 1 (Sections 1.1, 1.4, 1.6), Chapter 2 (Section 2.3), Chapter 3 (Sections 3.1, 3.2, 3.4).
[3]: Chapter 2.

Introduction to compartmental models, lake pollution model (with case study of Lake Burley Griffin), drug assimilation into the blood (case of a single cold pill, case of a course of cold pills, case study of alcohol in the bloodstream), exponential growth of population, limited growth of population, limited growth with harvesting, discrete population growth and chaos, logistic equation with time lag.

References:

[1]: Chapter 2 (Sections 2.1, 2.5-2.8), Chapter 3.

Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, method of undetermined coefficients, method of variation of parameters, application to projectile motion.

References:

[2]: Chapter 3 (Sections 3.3, 3.5-3.6).

Equilibrium points, interpretation of the phase plane, predator-prey model and its analysis, competing species and its analysis, epidemic model of influenza and its analysis, battle model and its analysis.

References:

[1]: Chapter 5 (Sections 5.3-5.4, 5.6-5.7), Chapter 6.

Power series solution about an ordinary point, solution about a singular point, Bessel's equation and Legendre's equation, Laplace transform and inverse transform, application to initial value problem up to second order.

References:

[2]: Chapter 7 (Sections 7.1-7.3), Chapter 8 (Sections 8.2-8.3).

Practical/ Lab work to be performed on a computer:

Modeling of the following problems using *Mathematica/ Maple/ Matlab*

(i) Plotting second and third order solution families
(ii) Growth and decay model
(iii) Lake pollution model
(iv) Case of a single cold pill and a course of cold pills
(v) Case study of alcohol in the bloodstream
(vi) Limited growth of population
(vii) Discrete population growth and chaos
(viii) Logistic equation with time lag
(ix) Automated variation of parameters
(x) Predator prey model
(xi) Epidemic model of influenza
(xii) Battle model

References:

1. Belinda Barnes and Glenn R. Fulford, *Mathematical Modeling with Case Studies, A Differential Equation Approach Using Maple*, Taylor and Francis, London and New York, 2002.

2. C. H. Edwards and D. E. Penny, *Differential Equations and Boundary Value Problems: Computing and Modeling*, Pearson Education, India, 2005.

3. S. L. Ross, *Differential Equations*, John Wiley and Sons, India, 2004.

Paper V: Analysis II

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Riemann integral, basic inequality of Riemann integral, Riemann condition of integrability, Riemann sum, algebraic and order properties of the Riemann integral. Riemann integrability for continuous functions, monotonic functions and functions with finite number of discontinuities, the fundamental theorem of calculus, consequences of the fundamental theorem of calculus: integration by parts and change of variables, mean value theorem of calculus.

References:

[4]: Chapter 6 (Articles 32-34).

Pointwise and uniform convergence of sequence of functions, uniform norm, uniform convergence and continuity, uniform convergence and differentiation, properties of exponential, logarithmic and trigonometric functions, arbitrary term series, rearrangement of series, series of functions, Weierstrass M-test, Weierstrass approximation theorem (statement only).

References:

[1]: Chapter 8 (Sections 8.1, 8.2.1-8.2.3, 8.3-8.4), Chapter 9 (Sections 9.1, 9.4.1-9.4.6).

Power series, radius of convergence, Cauchy-Hadamard theorem, improper integrals, convergence of improper integrals, tests of convergence for improper integrals, Abel's and Dirichlet's tests for improper integrals, Beta and Gamma functions and their relations.

References:

[1]: Chapter 9 (Sections 9.4.7-9.4.13)
[2]: Chapter 9 (Sections 9.4-9.6).

Iterated integrals (double and triple), integrals over rectangles and boxes, reduction to iterated integrals, properties of double integrals, double integrals over a region, mean value theorem for double integrals, change of variables for double and triple integrals, applications of multiple integrals to average value, volume, mass, centre of mass for regions in plane, moment of inertia.

References:

[3]: Chapter 5.

Definition of a line integral, differential form notation of a line integral, line integrals of gradient fields, independence of parametrization, line integrals along geometric curves, integrals of scalar functions along paths, parametrized surfaces, tangent plane, area of a surface, integral of a scalar function over a surface, surface integrals, orientation, surface integral for graphs, independence of parametrization, application to work, area of a shadow, fluid flow.

References:

[3]: Chapter 6.

Green's theorem, vector form of Green's theorem, Gauss' divergence theorem in the plane, Stoke's theorem, circulation and curl, Gauss' theorem, divergence and flux, Gauss' law, inter relations of Green's, Stoke's and Gauss' theorem, illustrations to demonstrate that they are higher dimensional version of the fundamental theorem of calculus.

References:

[3]: Chapter 7.

Use of computer aided software for example, Matlab/ Mathematica/ Maple/ MuPad wxMaxima for applications of multiple integrals to average value, volume, mass, centre of mass for regions in a plane, moment of inertia, work, area of a shadow, fluid flow.

References:

1. R. G. Bartle and D. R. Sherbert. *Introduction to Real Analysis* (3rd Edition), John Wiley and Sons (Asia) Pte. Ltd., Singapore. 2002.

2. Sudhir R. Ghorpade and Balmohan V. Limaye. *A Course in Calculus and Real Analysis.* Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2006.

3. J. E. Marsden, A. J. Tromba and A. Weinstein. *Basic multivariable calculus.* Springer (SIE), Indian reprint, 2005.

4. K. A. Ross. *Elementary analysis: the theory of calculus.* Undergraduate Texts in Mathematics, Springer (SIE). Indian reprint. 2004.

Paper VI: C++ Programming and Numerical Methods

Total marks: 100
Theory: 50
Practical: 25
Internal assessment: 25 (15 for theory and 10 for practicals)
4 Lectures, 2 Practicals (per week per student), 1 Tutorial (per fortnight per student)

Note: Maximum number of students allowed per batch for the practicals shall be 25. There will be one external examiner and one internal examiner for the practical examination. The duration of the practical examination will be 3 hours.

Introduction to structured programming: data types- simple data types, floating data types, character data types, string data types, arithmetic operators and operator precedence, variables and constant declarations, expressions, input using the extraction operator >> and cin, output using the insertion operator << and cout, preprocessor directives, increment (++) and decrement operations (--), creating a C ++ program, input/output, relational operators, logical operators and logical expressions, if and if … else statement, switch and break statements.

References:

[4]: Chapter 2 (pages 24-91). Chapter 3 (pages 96-137). Chapter 4 (pages 146-200).

"for", "while" and "do – while" loops, break and continue statement, nested control statement, value returning functions, void functions, value versus reference parameters, local and global variables, static and automatic variables, enumeration type, one dimensional array, two dimensional array, character array, pointer data and pointer variables.

References:

[4]: Chapter 5 (pages 204-265). Chapter 6. Chapter 7 (pages 273-368). Chapter 8 (pages 374-416). Chapter 9 (pages 424-465). Chapter 13 (pages 686-696).

Remark: The emphasis for C++ programming should be more on practicals rather than the theory.

Bisection method, false position method. Newton's method, secant method, rate of convergence, LU decomposition. Cholesky methods, error analysis, condition number, error estimate, ill conditioned system, Gauss-Jacobi, Gauss-Siedel and SOR iterative methods.
Development of programs for the above methods with applications to electrical circuits, input-output model for a simple economy.

References:

[1]: Chapter 1 (Section 1.2). Chapter 2 (Sections 2.1-2.2. 2.4-2.5). Chapter 3 (Sections 3.4-3.5, 3.7-3.8, pages 164-166).

Lagrange and Newton interpolation: linear and higher order. finite difference operators, interpolating polynomials using finite differences.
Development of programs for the above methods with applications to properties of water, spread of an epidemic.

References:

[3]: Chapter 4 (Sections 4.3, 4.4 up to Gregory Newton Backward interpolation).
[1]: Chapter 5 (Sections 5.1, 5.3, overview).

Numerical differentiation: forward difference, backward difference and central difference.
Integration: trapezoidal rule, Simpson's rule IVP of ODE: Euler's method.

References:

[1]: Chapter 6 (Sections 6.2, 6.4). Chapter 7 (Section 7.2).

Language programs and Numerical programs of the following (and similar) type have to be done:

i. Calculate the sum 1/1 + 1/2 + 1/3 + 1/4 + ----------+ 1/ N.
ii. Read floating point numbers and computes two averages: the average of negative numbers and the average of the positive numbers.
iii. Logical (i.e. Boolean) valued function which takes a single integer parameter and returns " True " if and only if the integer is a prime number between 1 and 1000.
iv. User defined function to find the absolute value of an integer.
v. Generate a random number between 0 and 99.
vi. Function "print...pyramid(..)" which takes a single integer argument "height" and displays a "pyramid" of this height made up of "*" characters on the screen.
vii. Using two dimensional arrays, write c ++ function (and a corresponding program to test it) which multiplies an m x n matrix of integers by an n x r matrix of integers.
viii. To create employee data base using two dimensional arrays.
ix. Enter 100 integers into an array and sort them in an ascending order.

x. Enter 10 integers into an array and then search for a particular integer in the array.

xi. Read from a text file and write to a text file.

xii. Demonstrate the use of enumeration.

xiii. Root finding methods - Newton and secant Method.

xiv. LU decomposition and Cholesky methods.

xv. Gauss-Jacobi and Gauss-Siedel method.

xvi. Lagrange and Newton Interpolation

xvii. Gregory Newton Backward and forward interpolation.

xviii. Simpson's rule.

xix. Euler's method.

Some part of the project work might consist of the programs of the following type:

- Find roots of a second degree polynomial.
- Enter 10 integers into an array and print the largest of them.
- Bisection, false position.
- SOR iterative methods.
- Trapezoidal rule.

Reference:

1. B. Bradie, *A friendly introduction to Numerical Analysis*, Pearson Education, India, 2006.

2. C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, Pearson Education, India, 2005.

3. M. K. Jain, S. R. K. Iyengar and R. K. Jain, *Numerical methods for scientific and engineering computation*, New age International Publisher, India, 2003.

4. D. S. Malik, *C++ Programming: Program design including data structures* (2nd Edition), Thomson Course Technology, Thomson Press, USA, 2004.

Paper VII: Algebra II

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, a Check-Digit Scheme based on the dihedral group D_5, product (HK) of two subgroups, definition and properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem, an application of cosets to permutation groups, the rotation group of a cube and a soccer ball; definition and examples of the external direct product of a finite number of groups, normal subgroups, factor groups, applications of factor groups to the alternating group A_4, commutator subgroup.

References:

[2]: Chapter 5, Chapter 7 (including Exercises 3, 6 and 7 on page 168), Chapter 8 (till the end of Example 2), Chapter 9 (till the end of Example 13 and including Exercise 52 on page 188).

Definition and examples of homomorphisms, properties of homomorphisms, definition and examples of isomorphisms, Cayley's theorem, properties of isomorphisms, Isomorphism theorems I, II and III, definition and examples of automorphisms, inner automorphisms, automorphism and inner automorphism group, automorphism group of finite and infinite cyclic groups, applications of factor groups to automorphism groups, Cauchy's theorem for finite abelian groups.

References:

[2]: Chapter 6, Chapter 9 (Theorems 9.3-9.5), Chapter 10.

Characteristic of a ring, ideals, ideal generated by subsets in a commutative ring with unity, factor rings, operations on ideals, prime ideals and maximal ideals, definition and examples of ring homomorphisms, properties of ring homomorphisms, isomorphisms, Isomorphism theorems I, II and III, field of quotients.

References:

[2]: Chapter 13, Chapter 14, Chapter 15.

Definition of polynomial rings over commutative rings, the division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, unique factorization in Z[x], an application of unique factorization to weird dice, divisibility in integral domains, irreducibles, primes, unique factorization domains, Euclidean domains.

References:

[2]: Chapter 16, Chapter 17, Chapter 18.

Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combinations and systems of linear equations, linear span, linear independence, basis and dimension, dimensions of subspaces, linear transformations, null space, range, rank and nullity of linear transformations, matrix of a linear transformation, algebra of linear transformations, isomorphism, Isomorphism theorems, invertibility and isomorphisms, change of basis.

References:

[1]: Chapter 1 (Sections 1.2-1.6), Chapter 2 (Sections 2.1-2.5).

Dual spaces, dual basis, double dual, transpose and its matrix in the dual basis, annihilators, eigenvalues and eigenvectors, characteristic polynomial, diagonalizability, invariant subspaces and the Cayley-Hamilton theorem, the minimal polynomial for a linear transformation.

References:

[1]: Chapter 2 (Section 2.6), Chapter 5 (Sections 5.1-5.2, 5.4, 7.3).

References:

1. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, *Linear Algebra* (4th Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.

2. Joseph A. Gallian, *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi, 1999.

Paper VIII: Differential Equations and Mathematical Modeling II

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Introduction, classification, construction and geometrical interpretation of first order partial differential equations (PDE), method of characteristic and general solution of first order PDE, canonical form of first order PDE, method of separation of variables for first order PDE.

References:

[2]: Chapter 2.

Mathematical modeling of vibrating string, vibrating membrane, conduction of heat in solids, gravitational potential, conservation laws and Burger's equations, classification of second order PDE, reduction to canonical forms, equations with constant coefficients, general solution.

References:

[2]: Chapter 3 (Sections 3.1-3.3, 3.5-3.7), Chapter 4.

Cauchy problem for second order PDE, homogeneous wave equation, initial boundary value problems, non-homogeneous boundary conditions, finite strings with fixed ends, non-homogeneous wave equation, Riemann problem, Goursat problem, spherical and cylindrical wave equation.

References:

[2]: Chapter 5.

Method of separation of variables for second order PDE, vibrating string problem, existence and uniqueness of solution of vibrating string problem, heat conduction problem, existence and uniqueness of solution of heat conduction problem, Laplace and beam equation, non-homogeneous problem.

References:

[2]: Chapter 7.

Monte Carlo Simulation Modeling: simulating deterministic behavior (area under a curve, volume under a surface). Generating Random Numbers: middle square method, linear congruence. Queuing Models: harbor system, morning rush hour. Linear Programming Model: algebraic solution, simplex method.

References:

[3]: Chapter 5 (Sections 5.1-5.2), Chapter 7 (Sections 7.3-7.5).

Graphs, diagraphs, networks and subgraphs, vertex degree, paths and cycles, regular and bipartite graphs, four cube problem, social networks, exploring and traveling, Eulerian and Hamiltonian graphs, applications to dominoes, diagram tracing puzzles, Knight's tour problem, gray codes.

References:

[1]: Chapter 1 (Section 1.1), Chapter 2, Chapter 3.

Remark: **Chapter 1 (Section 1.1), Chapter 2 (Sections 2.1-2.4), Chapter 3 (Sections 3.1-3.3) are to be reviewed only. This is in order to understand the models on Graph Theory.**

Using Computer aided software for example, Matlab/ Mathematica/ Maple/ MuPad- characteristics, vibrating string, vibrating membrane, conduction of heat in solids, gravitational potential, conservation laws and Burger's equations, area under a curve, generating random numbers, morning rush hour.

References:

1. Joan M. Aldous and Robin J. Wilson, *Graphs and Applications: An Introductory Approach*, Springer, Indian reprint, 2007.

2. Tyn Myint-U and Lokenath Debnath, *Linear Partial Differential Equation for Scientists and Engineers*, Springer, Indian reprint, 2006.

3. Frank R. Giordano, Maurice D. Weir, William P. Fox, *A First Course in Mathematical Modeling*, Thomson Learning, London and New York, 2003.

Chebyshev's inequality. statement and interpretation of (weak) law of large numbers and strong law of large numbers. Central Limit theorem for independent and identically distributed (iid) random variables with finite variance. Markov Chains, Chapman-Kolmogorov equations. classification of states, sampling distributions of sample mean, definitions and simple properties of chi-square, t and F distributions.

References:

[2]: Chapter 4 (Section 4.4), Chapter 8 (Sections 8.1-8.6).
[3]: Chapter 2 (Section 2.7), Chapter 4 (Sections 4.1-4.3).

Unbiased estimation, minimum variance unbiased estimators and Cramer-Rao inequality, methods of estimation: maximum likelihood and moments, interval estimation, estimation of means and difference in means, estimation of proportions and difference between proportions, estimation of variances and the ratio of two variances.

[2]: Chapter 10 (Sections 10.1-10.3, 10.7-10.8), Chapter 11 (Sections 11.1-11.7).

Testing of Hypothesis: simple and composite hypotheses, null and alternative hypotheses, type-I and type-II errors, size and power of the test, Z and t-test for single mean, Z and t-test for difference in means, test concerning proportions, test for difference in proportions, chi-square and F-tests concerning variances, the analysis of an r x c table, chi-square test for goodness of fit.

References:

[2]: Chapter 12 (Sections 12.1-12.2, 12.5), Chapter 13 (Sections 13.1-13.5, 13.7-13.8, Exercise 13.12).

Practical/ Lab work to be performed on a computer using SPSS/Excel.

Practicals should broadly cover the following areas:

(i) Fitting of Binomial, Poisson, Negative Binomial, Normal Distributions.
(ii) Applications of Chi-square, t and F Distributions.
(iii) Calculation of correlation coefficient, Rank Correlation, etc.
(iv) Fitting of polynomials and regression curves.

(iv) Methods of estimation (MLE and method of Moments)

(v) Selecting a simple random sample using random number tables.

References:

1. Robert V. Hogg, Joseph W. McKean and Allen T. Craig, *Introduction to mathematical statistics*, Pearson Education, Asia, 2007.

2. Irwin Miller and Marylees Miller, *John E. Freund's mathematical statistics with applications* (7th Edition), Pearson Education, Asia, 2006.

3. Sheldon Ross, *Introduction to probability models* (9th Edition), Academic Press, Indian Reprint, 2007.

Paper X: Algebra III

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Properties of external direct products, the group of units modulo n as an external direct product, applications of external direct products to data security, public key cryptography, definition and examples of internal direct products, fundamental theorem of finite abelian groups, definition and examples of group actions, stabilizers and kernels of group actions, permutation representation associated with a given group action.

References:

[1]: Chapter 1 (Section 1.7), Chapter 2 (Section 2.2), Chapter 4 (Section 4.1).
[3]: Chapter 8, Chapter 9 (Section on internal direct products), Chapter 11.

Applications of group actions: Cauchy's theorem, Index theorem, Cayley's theorem, conjugacy relation, class equation and consequences, conjugacy in S_n, p-groups, Sylow's theorems and consequences.
Definition and examples of simple groups, non-simplicity tests, composition series, Jordan-Holder theorem, solvable groups.

References:

[1]: Chapter 3 (Section 3.4, Exercise 9), Chapter 4 (Sections 4.2-4.3, 4.5-4.6).
[3]: Chapter 25.

Extension fields, fundamental theorem of field theory, splitting fields, zeros of an irreducible polynomial, perfect fields, characterization of extensions, algebraic extensions, finite extensions, properties of algebraic extensions, classification of finite fields, structure of finite fields, subfields of finite fields, constructible numbers, straight edge and compass construction.

References:

[3]: Chapter 20, Chapter 21, Chapter 22, Chapter 23.

Inner product spaces and norms. Gram-Schmidt orthogonalisation process. orthogonal complements. Bessel's inequality. adjoints of linear operators. least square approximations, minimal solutions to system of linear equations.

References:

[2]: Chapter 6 (Sections 6.1-6.3).

Normal operators and self-adjoint operators, unitary and orthogonal operators, matrices of orthogonal and unitary operators, rigid motions, orthogonal operators on R^2, conic sections.

References:

[2]: Chapter 6 (Sections 6.4-6.5).

Primary decomposition theorem, theorem on decomposition into sum of diagonalizable and nilpotent operator, cyclic subspaces and annihilators, cyclic decomposition theorem, rational form, invariant factors, Jordan form.

References:

[4]: Chapter 6 (Section 6.8), Chapter 7 (Sections 7.1-7.3).

References:

1. David S. Dummit and Richard M. Foote, *Abstract Algebra* (2nd Edition), John Wiley and Sons (Asia) Pte. Ltd. Singapore, 2003.

2. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence. *Linear Algebra* (4th Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.

3. Joseph A. Gallian. *Contemporary Abstract Algebra* (4th Edition), Narosa Publishing House, New Delhi. 1999.

4. Kenneth Hoffman and Ray Kunze. *Linear Algebra* (2nd edition), Pearson Education Inc., India. 2005.

Paper XI: Analysis III

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Definition and examples of metric spaces, isometries, diameter, isolated points, accumulation and boundary points, closure and interior, open and closed sets, Cantor's intersection theorem, open and closed balls, convergence, Cauchy sequence and boundedness.

References:

[2]: Chapter 1 (Sections 1.1 (up to example 1.1.17), 1.3-1.4, 1.6-1.7), Chapter 2 (Sections 2.1-2.3, 2.5-2.6), Chapter 3 (Sections 3.1, 3.6-3.7), Chapter 4 (Sections 4.1-4.4, 4.7), Chapter 5 (Sections 5.1-5.3), Chapter 6 (Sections 6.1-6.2, 6.4-6.8), Chapter 7 (Sections 7.1, 7.4, 7.6-7.8).

Continuity and uniform continuity, completeness, contraction mapping theorem, open covers, definition of a compact set (using open covers), criteria of compactness and their equivalence.

References:

[2]: Chapter 8 (Sections 8.1-8.3, 8.5, 8.9-8.10), Chapter 9 (Sections 9.1 (up to Subsection 9.1.3), 9.2 (Theorem 9.2.1 with 1st two criteria), 9.4, 9.9), Chapter 10 (Sections 10.2 (only Cauchy criterion), 10.3, 10.8, 10.10), Chapter 12 (Sections 12.1 (excluding criteria (iv),(v) and (viii) in the theorem 12.1.3), 12.3, 12.5).

Review of complex plane, sequences and series, connected sets and polygonally connected sets in the complex plane, stereographic projection, analytic polynomials, power series, analytic functions, Cauchy-Riemann equations, functions e^z, $\sin z$, and $\cos z$.

References:

[1]: Chapter 1, Chapter 2, Chapter 3.

...ne integrals and their properties, closed curve theorem for entire functions, Cauchy integral formula and Taylor expansions for entire functions, Liouville's theorem and the fundamental theorem of algebra.

References:

[1]: Chapter 4, Chapter 5.

Power series representation for functions analytic in a disc, analyticity in an arbitrary open set, uniqueness theorem, definitions and examples of conformal mappings, bilinear transformations.

References:

[1]: Chapter 6 (Sections 6.1-6.2, 6.3 (up to theorem 6.9), Chapter 9 (Sections 9.2, 9.7-9.8, 9.9 (statement only), 9.10, 9.11 (with examples), 9.13), Chapter 13 (Sections 13.1, 13.2 (up to Theorem 13.11 including examples).

Piecewise continuous functions, Fourier cosine and sine series, Fourier series, property of Fourier coeffcients, Fourier theorem, discussion of the theorem and its corollary.

References: [3].

Use of computer aided software for example, Matlab/ Mathematica/ Maple/ MuPad/ Maxima for power series of analytic functions, Fourier series.

References:

1. Joseph Bak and Donald J. Newman, *Complex analysis* (2^{nd} Edition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.

2. Mícheál Ó Searcóid, *Metric Spaces*, Springer Undergraduate Mathematics Series, Springer-Verlag London Limited, London, 2007.

3. *Fourier Series*, lecture notes published by the Institute of Life Long Learning, University of Delhi, Delhi, 2008.

Paper XII (i) (Optional): Applications of Algebra

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures, 1 Tutorial (per week per student)

Balanced incomplete block designs (BIBD): definitions and results, incidence matrix of BIBD, construction of BIBD from difference sets, construction of BIBD using quadratic residues, difference set families, construction of BIBD from finite fields.

References:

[2]: Chapter 2 (Sections 2.1-2.4, 2.6).

Coding theory: introduction to error correcting codes, linear codes, generator and parity check matrices, minimum distance, Hamming Codes, decoding, and cyclic codes.

References:

[2]: Chapter 4 (Sections 4.1-4.3, up to Example 4.3.17).

Symmetry groups and color patterns: review of permutation groups, groups of symmetry and action of a group on a set; colouring and colouring patterns, Polya theorem and pattern inventory, generating functions for non-isomorphic graphs.

References:

[2]: Chapter 5.

Special types of matrices: idempotent, nilpotent, involution, and projection tridiagonal matrices, circulant matrices, Vandermonde matrices, Hadamard matrices, permutation and doubly stochastic matrices. Frobenius- König theorem, Birkhoff theorem.

References:

[5]: Chapter 4.

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Positive Semi-definite matrices: positive semi-definite matrices, square root of a positive semi-definite matrix, a pair of positive semi-definite matrices, and their simultaneous diagonalization.

Symmetric matrices and quadratic forms: diagonalization of symmetric matrices, quadratic forms, constrained optimization, singular value decomposition, and applications to image processing and statistics.

References:

[5]: Chapter 6 (Sections 6.1-6.2).
[4]: Chapter 7.

Applications of linear transformations: Fibonacci numbers, incidence models, and differential equations.

Least squares methods: Approximate solutions of system of linear equations, approximate inverse of an m×n matrix, solving a matrix equation using its normal equation, finding functions that approximate data.

Linear algorithms: LDU factorization, the row reduction algorithm and its inverse, backward and forward substitution, approximate inverse and projection algorithms.

References:

[1]: Chapter 9 (Sections 9.1, 9.4-9.5), Chapter 10, Chapter 11.

The lectures for the topics BIBD, coding theory, symmetry groups and colour patterns, and the last set of topics include the use of computer aided software for example Matlab/ Mathematica/ Maple/ MuPad/ wxMaxima to demonstrate

(i) *construction of Hadamard matrices,*
(ii) *construction of Linear (Hamming) codes,*
(iii) *construction of the initial blocks and corresponding block designs,*
(iv) *construction of pattern inventories and how to count patterns,*
(v) *how to test the algorithms.*

Maple routines for some of the above can be found in [3].

References:

1. I. N. Herstein and D. J. Winter. *Primer on Linear Algebra*. Macmillan Publishing Company, New York, 1990.

Positive Semi-definite matrices: positive semi-definite matrices, square root of a positive semi-definite matrix, a pair of positive semi-definite matrices, and their simultaneous diagonalization.

Symmetric matrices and quadratic forms: diagonalization of symmetric matrices, quadratic forms, constrained optimization, singular value decomposition, and applications to image processing and statistics.

References:

[5]: Chapter 6 (Sections 6.1-6.2).
[4]: Chapter 7.

Applications of linear transformations: Fibonacci numbers, incidence models, and differential equations.

Least squares methods: Approximate solutions of system of linear equations, approximate inverse of an m×n matrix, solving a matrix equation using its normal equation, finding functions that approximate data.

Linear algorithms: LDU factorization, the row reduction algorithm and its inverse, backward and forward substitution, approximate inverses and projection algorithms.

References:

[1]: Chapter 9 (Sections 9.1, 9.4-9.5), Chapter 10, Chapter 11.

The lectures for the topics BIBD, coding theory, symmetry groups and colour patterns, and the last set of topics include the use of computer aided software for example Matlab/ Mathematica/ Maple/ MuPad/ wxMaxima to demonstrate

(i) *construction of Hadamard matrices,*
(ii) *construction of Linear (Hamming) codes,*
(iii) *construction of the initial blocks and corresponding block designs,*
(iv) *construction of pattern inventories and how to count patterns,*
(v) *how to test the algorithms.*

Maple routines for some of the above can be found in [3].

References:

1. I. N. Herstein and D. J. Winter. *Primer on Linear Algebra*. Macmillan Publishing Company, New York, 1990.

2. S. R. Nagpaul and S. K. Jain. *Topics in Applied Abstract Algebra*. Thomson Brooks and Cole. Belmont, 2005.

3. Richard E. Klima, Neil Sigmon, Ernest Stitzinger, *Applications of Abstract Algebra with Maple*, CRC Press LLC, Boca Raton, 2000.

4. David C. Lay, *Linear Algebra and its Applications* (3rd Edition). Pearson Education Asia, Indian Reprint, 2007.

5. Fuzhen Zhang, *Matrix theory*, Springer-Verlag New York, Inc., New York, 1999.

Paper XII (ii) (Optional): Discrete Mathematics

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures, 1 Tutorial (per week per student).

Definition, examples and basic properties of ordered sets, maps between ordered sets, duality principle, down-sets, up-sets, maximal and minimal elements, building new ordered sets, lattices as ordered sets, complete lattices, lattices as algebraic structures, sublattices, products and homomorphisms.

References:

[1]: Chapter 1, Chapter 2 (Sections 2.1-2.17), Chapter 5 (Sections 5.1-5.11).
[3]: Chapter 1 (Section 1).

Definition, examples and properties of modular and distributive lattices, Boolean algebras, Boolean polynomials, ideals, filters and equations, minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching circuits and applications of switching circuits.

References:

[1]: Chapter 6.
[3]: Chapter 1 (Sections 3-6), Chapter 2 (Sections 7-8).

Definition, examples and basic properties of graphs, pseudographs, complete graphs, bi-partite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm, Floyd-Warshall algorithm.

References:

[2]: Chapter 9, Chapter 10.

Applications of paths and circuits: the Chinese postman problem, digraphs, the Bellman-Ford algorithm, tournaments, directed network, scheduling problems, definitions, examples and basic properties of trees, spanning trees, minimum spanning tree algorithms, Kruskal's algorithm, Prim's algorithm, acyclic digraphs, Bellman's algorithm.

References:

[2]: Chapter 11 (Sections 11.1-11.2, 11.4-11.5), Chapter 12.

Planar graphs, colouring of graphs, statement of the four-colour theorem, the five-colour theorem, circuit testing, facilities design, flows and cuts, construction of flows, constructing maximal flows, rational weights, applications of directed networks, matchings.

References:

[2]: Chapter 14, Chapter 15.

Discrete numeric functions, generating functions, recurrence relations, linear recurrence relations with constant coefficients, homogenous solutions, particular solutions, total solutions, solution by method of generating functions.

References:

[4]: Chapter 9 (Sections 9.1-9.2, 9.4), Chapter10 (Sections 10.1-10.7).

References:

1. B A. Davey and H. A. Priestley, *Introduction to Lattices and Order*, Cambridge University Press, Cambridge, 1990.

2. Edgar G. Goodaire and Michael M. Parmenter, *Discrete Mathematics with Graph Theory* (2nd Edition), Pearson Education (Singapore) Pte. Ltd., Indian Reprint, 2003.

3. Rudolf Lidl and Günter Pilz, *Applied Abstract Algebra* (2nd Edition), Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.

4. C. L. Liu, *Elements of Discrete Mathematics* (2nd Edition), Tata McGraw Hill Publishing Company Limited, New Delhi, 2001.

Paper XII (iii) (Optional): Mathematical Finance

Total marks: 100
Theory: 75
Internal assessment: 25
4 Lectures, 1 Practical, 1 Tutorial (per student per week)

Basic principles: comparison, arbitrage and risk-aversion. Interest (simple and compound, discrete and continuous), time value of money, inflation, net present value, internal rate of return (calculation by bisection and Newton-Raphson methods), comparison of NPV and IRR.
Bonds, bond prices and yields, Macaulay and modified duration, term structure of interest rates: spot and forward rates, explanations of term structure, running present value, floating-rate bonds, Fisher-Weil and Quasi-modified duration, immunization, convexity, putable and callable bonds.

References:

[2]: Chapter 1, Chapter 2, Chapter 3, Chapter 4.

Asset return, short selling, portfolio return, (brief introduction to expectation, variance, covariance and correlation), random returns, portfolio mean return and variance, diversification, portfolio diagram, feasible set, Markowitz model (review of Lagrange multipliers for 1 and 2 constraints), Two fund theorem, risk free assets, One fund theorem, capital market line, Sharpe index.
Capital Asset Pricing Model (CAPM), betas of stocks and portfolios, security market line, use of CAPM in investment analysis and as a pricing formula, Jensen's index, Harmony theorem, data and statistics.

References:

[2]: Chapter 6, Chapter 7, Chapter 8 (Sections 8.5-8.8).
[4]: Chapter 1 (for a quick review/description of expectation etc.)

Forwards and futures, marking to market, value of a forward/futures contract, replicating portfolios, futures on assets with known income or dividend yield, currency futures, hedging (short, long, cross, rolling), optimal hedge ratio, hedging with stock index futures, interest rate futures, swaps.

Lognormal distribution. Lognormal model/ Geometric Brownian Motion for stock prices. Binomial Tree model for stock prices, parameter estimation, comparison of the models.

References:

[2]: Chapter 10 (except 10.11, 10.12), Chapter 11 (except 11.2 and 11.8)
[3]: Chapter 3, Chapter 5, Chapter 6, Chapter 7 (except 7.10 and 7.11)
[4]: Chapter 3

European and American options, factors influencing options premiums, bounds on options premiums, put-call parity, Binomial option pricing model (BOPM), dynamic hedging, pricing American and exotic options with BOPM, risk neutral valuation (for European derivatives on assets following binomial tree model).
Extension of risk neutral valuation to assets following GBM (without proof), Black-Scholes formula for European options, implied volatility, hedging parameters (the "Greeks": delta, gamma, theta, rho and vega), dynamic hedging.

References:

[1]: Chapter 12 (except 12.8), Chapter 13 (except 13.2, 13.6, 13.7)
[3]: Chapter 9, Chapter 11, Chapter 12, Chapter 13 (except 13.10), Chapter 15 (Sections 15.1 to 15.9).
[4]: Chapter 7, Chapter 9.

Base Black-Scholes formula proof on Hull [3].

Speculation strategies (Spreads, Butterflies, Straddles, Strangles), Value at Risk (VaR), linear and quadratic models, Monte Carlo techniques (including an introduction to random number generation), use of Monte Carlo techniques to price exotic options and calculate VaR.

References:

[3]: Chapter 10, Chapter 18
[4]: Chapter 9

Practical/ Lab work to be performed on a computer using Excel.

The sample practicals below can be implemented using the standard features of Excel (functions, Solver, Goal Seek), without any use of macros/VBA. A choice of 10 practicals can be attempted from this list. The practicals should be of 2 hours each, held fortnightly.

i. Introduction to spreadsheet programs.
ii. Bonds and duration: computing PV, I. R. YTM, duration; immunization.
iii. Portfolio models: calculating portfolio means and variances.
iv. Calculating the efficient frontier and finding the capital market line with and without short sale restrictions.
v. Estimating betas and the Security Market Line.
vi. Testing CAPM.
vii. Simulating lognormal price paths.
viii. Calculating the parameters of the lognormal distribution from stock prices.
ix. Implementing the Black-Scholes formulae in a spreadsheet.
x. Calculating implied volatility.
xi. Hedging and portfolio insurance.
xii. VaR estimation by Monte Carlo methods.

References:

1. Simon Benninga, *Financial Modeling*, MIT Press, Cambridge, Massachusetts, 1997.

2. David G Luenberger, *Investment Science*, Oxford University Press, Delhi, 1998.

3. John C Hull, *Options, Futures and Other Derivatives* (6th Edition), Prentice-Hall India, Indian reprint, 2006.

4. Sheldon Ross, *An elementary Introduction to Mathematical Finance* (2nd Edition), Cambridge University Press, USA, 2003.

Paper XII (iv) (Optional): Number Theory and Cryptography

Total marks: 100
Theory: 75
Internal assessment: 25
5 Lectures. 1 Tutorial (per week per student)

Division algorithm. Lame's theorem. linear Diophantine equation. fundamental theorem of arithmetic. prime counting function. statement of prime number theorem. sieve of Eratosthenes. Goldbach conjecture. binary and decimal representation of integers. linear congruences. complete set of residues. Chinese remainder theorem. polynomial congruences. Fermat's little theorem. pseudoprimes. Wilson's theorem, Fermat-Kraitchik factorization method.

References:

[1]: Chapter 2 (Sections 2.2-2.5). Chapter 3. Chapter 4. Chapter 5.
[3]: Chapter 2 (Theorem 2.4). *Chapter 3 (Section 3.2). Chapter 4 (Section 4.5).*

Number theoretic functions. sum and number of divisors. totally multiplicative functions. definition and properties of the Dirichlet product. the Möbius inversion formula. the greatest integer function. Euler's phi-function. Euler's theorem. reduced set of residues. some properties of Euler's phi-function.

References:

[1]: Chapter 6 (Sections 6.1-6.3). Chapter 7.
[3]: Chapter 5 (Section 5.2 (Definition 5.5-Theorem 5.40. Section 5.3 (Theorem 5.15-Theorem 5.17, Theorem 5.19)).

Order of an integer modulo n, primitive roots for primes. composite numbers having primitive roots. theory of indices. Euler's criterion. the Legendre symbol and its properties. quadratic reciprocity. quadratic congruences with composite moduli. Jacobi symbol and its properties.

References:

[1]: Chapter 8. Chapter 9.

[3]: Chapter 7 (Section 7.6).

Perfect numbers. Mersenne primes and amicable numbers. Fermat numbers. the equation $x^2 + y^2 = z^2$. Fermat's last theorem. sums of two squares. sums of more than two squares. the Fibonacci sequence. certain identities involving Fibonacci numbers.

References:

[1]: Chapter 11. Chapter 12. Chapter 13. Chapter 14.

Remark: ***For the topics that follow on cryptography there should be emphasis on implementation of the encryption routines.***

Elementary cryptosystems. Hill cryptosystem, generalization of the Hill cryptosystem. public key encryption. RSA encryption and decryption, notes on primality testing. integer factorization and digital signatures.

References:

[1]: Chapter 10 (Section 10.1).
[2]: Chapter 6, Chapter 7 (Sections 7.2-7.3, 7.5-7.7).

Diffie Hellman key exchange. the knapsack cryptosystem. the ElGamal cryptosystem. elliptic curves, elliptic curve cryptography through examples, a discussion of some unsolved problems in number theory.

References:

[1]: Chapter 10 (Sections 10.2-10.3).
[2]: Chapter 7 (Section 7.8). Chapter 8 (Sections 8.1-8.4. Section 8.5 (Example 8.4)).
[3]: Appendix A.

The lectures for the topics on cryptography should include implementation of the encryption and decryption routines using computer aided software such as Matlab/ Mathematica/ Maple/ MuPad/ wxMaxima.

References:

1. David M. Burton. *Elementary Number Theory* (6th Edition). Tata McGraw-Hill Edition, Indian reprint, 2007.

2. Richard E. Klima, Neil Sigmon, Ernest Stitzinger, *Applications of Abstr*
Algebra with Maple, CRC Press, Boca Raton, 2000.

3. Neville Robinns, *Beginning Number Theory* (2[nd] Edition), Narosa Publish
House Pvt. Limited, Delhi, 2007.

Paper XII (v) (Optional): Optimization

Total marks: 100
Theory: 70
Internal assessment: 30 (15 for theory and 15 for practical)
5 Lectures, 1 Practical, 1 Tutorial (per week per student)

Introduction to optimization problems with examples including calculus of variations and linear programming problem (LPP), analytical methods applicable to optimization problems with constraints- direct substitution, constrained variation, Lagrange multipliers, method of steepest ascent, economic interpretation of the Lagrange multipliers, inequality constraints.

References:

[3]: Chapter 1, Chapter 2 (Section 2.2).

Calculus of Variations: Euler equation, functions, functionals, neighborhoods, functional with higher order derivatives in the integrand, functional with several functions in the integrand, functional with several functions and higher derivatives, functional with more then one independent variable, algebraic constraints, integral constraints, differential equation constraints.

References:

[3]: Chapter 8.

Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison.

References:

[1]: Chapter 3 (Sections 3.2-3.3, 3.5-3.8). Chapter 4 (Sections 4.1-4.4).

Duality, formulation of the dual problem, primal- dual relationships, economic interpretation of the dual, sensitivity analysis.

References:

[1]: Chapter 6 (Sections 6.1- 6.5, 6.7 (up to example 6.14)).

Transportation problem and its mathematical formulation, northwest-corner method least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.
Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

References:

[4]: Chapter 5 (Sections 5.1, 5.3-5.4).
[2]: Chapter 14.

Practical/ Lab work to be performed on a computer.

Case Studies of the following type and similar nature using computer software like Mu-Pad / Excel- Solver / LINGO / LINDO etc.

i. Assigning students to school
ii. New frontiers
iii. Controlling air pollution
iv. Farm management
v. Shipping wood to market
vi. Project pickings

References:

[2]: Chapter 4 (Cases 4.2, 4.3), Chapter 6 (Case 6.1, 6.2), Chapter 8 (Cases 8.1, 8.2).

References:

1. Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, *Linear programming and Network Flows* (2nd edition). John Wiley and Sons, India. 2004.

2. F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research* (8th Edition). Tata McGraw Hill, Singapore. 2004.

3. Ralph W. Pike, *Optimization for Engineering Systems*. Copyright 2001, Electronic book, http://www.ici.ro/camo/books/elbooks.htm.

4. Hamdy A. Taha, *Operations Research, An Introduction* (8th edition). Prentice-Hall India. 2006.